A Level Mathematics A

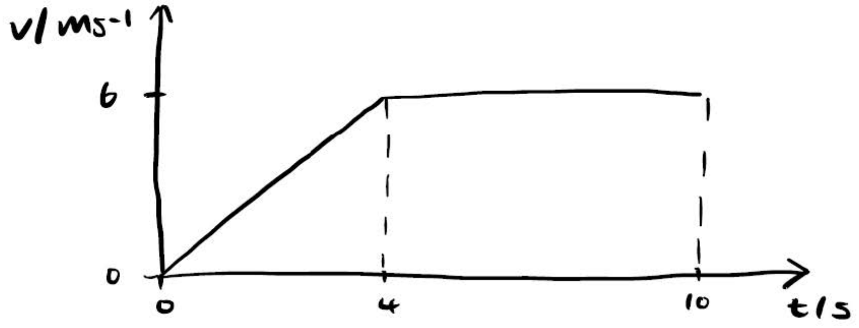
H240/03 Pure Mathematics and Mechanics

Question Set 4

1. A cyclist starting from rest accelerates uniformly at $1.5 \mathrm{~m~s^{-2}}$ for $4 \mathrm{~s}$ and then travels at constant speed.

(a) Sketch a velocity-time graph to represent the first 10 seconds of the cyclist's motion. [2]

$u = 0 \mathrm{ms^{-1}}$ $\quad a = 1.5 \mathrm{ms^{-2}}$ $\quad t = 4\mathrm{s}$ $\quad$ constant speed



(b) Calculate the distance travelled by the cyclist in the first 10 seconds. [2]

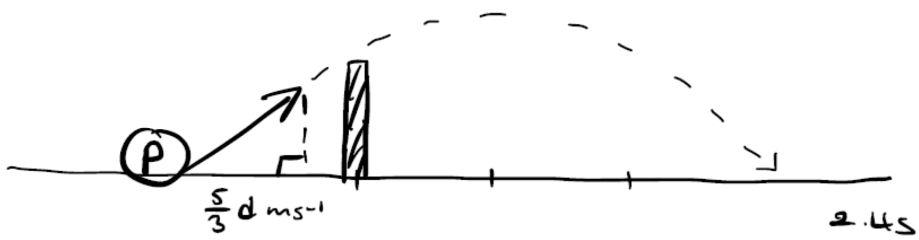
distance = area under graph

$= \frac{1}{2}(6\mathrm{ms^{-1}})(4\mathrm{s}) + (6\mathrm{ms^{-1}})(6\mathrm{s}) = 48\mathrm{m}$

2 A particle P projected from a point O on horizontal ground hits the ground after 2.4 seconds.

The horizontal component of the initial velocity of P is $\frac{5}{3}d \mathrm{~m~s^{-1}}$.

(a) Find, in terms of d, the horizontal distance of P from O when it hits the ground. [1]



$s = \frac{D}{t}$ $\quad \frac{5}{3}d \mathrm{ms^{-1}} = \frac{D}{2.4\mathrm{s}}$ $\quad D = 4d \mathrm{~m}$

(b) Find the vertical component of the initial velocity of P. [2]

u_v $\quad t = 1.2\mathrm{s}$ $\quad v = 0\mathrm{ms^{-1}}$ $\quad a = -9.81\mathrm{ms^{-2}}$

$v = u + at$ $\quad 0\mathrm{ms^{-1}} = u_v + (-9.81\mathrm{ms^{-2}})(1.2\mathrm{s})$

$u_v = 11.8\mathrm{ms^{-1}}$

P just clears a vertical wall which is situated at a horizontal distance d m from O.

(c) Find the height of the wall. **[3]**

$s = \frac{D}{t}$ $\quad \frac{5}{3}d\ \text{ms}^{-1} = \frac{d\text{m}}{t}$ $\quad t = \frac{3}{5}\text{s}$

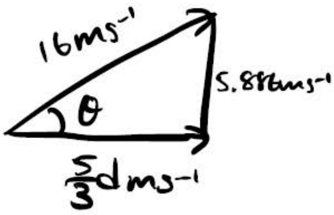
$u = 11.8\text{ms}^{-1}$ $\quad t = \frac{3}{5}\text{s}$ $\quad a = -9.81\text{ms}^{-2}$ $\quad s = ?$

$s = ut + \frac{1}{2}at^2$ $\quad s = (11.8\text{ms}^{-1})(\frac{3}{5}\text{s}) + \frac{1}{2}(-9.81\text{ms}^{-2})(\frac{3}{5}\text{s})^2$

$= \boxed{5.30\text{m}}$

The speed of P as it passes over the wall is 16ms^{-1}.

(d) Find the value of d correct to 3 significant figures. **[4]**



$u = 11.8\text{ms}^{-1}$ $\quad t = \frac{3}{5}\text{s}$ $\quad a = -9.81\text{ms}^{-2}$ $\quad v = ?$

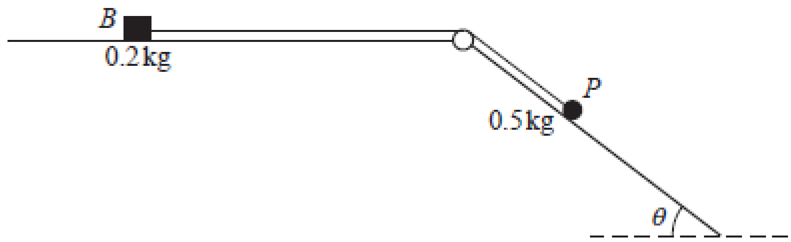
$v = u + at$ $\quad v = 11.8\text{ms}^{-1} + (-9.81\text{ms}^{-2})(\frac{3}{5}\text{s})$

$= 5.886\text{ms}^{-1}$

$\sin\theta = \frac{5.886}{16}$ $\quad \theta = 21.6°$

$\cos 21.6' = \frac{\frac{5}{3}d}{16}$ $\quad d = \boxed{8.93}$

3

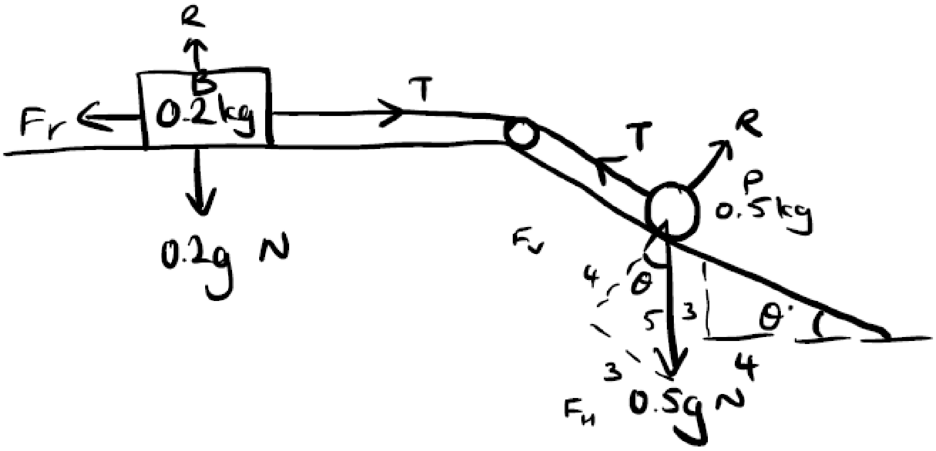


The diagram shows a small block B, of mass 0.2 kg, and a particle P, of mass 0.5 kg, which are attached to the ends of a light inextensible string. The string is taut and passes over a small smooth pulley fixed at the intersection of a horizontal surface and an inclined plane.

The block can move on the horizontal surface, which is rough. The particle can move on the inclined plane, which is smooth and which makes an angle of θ with the horizontal where $\tan\theta = \frac{3}{4}$.

The system is released from rest. In the first 0.4 seconds of the motion P moves 0.3 m down the plane and B does not reach the pulley.

(a) Find the tension in the string during the first 0.4 seconds of the motion. [4]



4s
0.3m

$u = 0ms^{-1}$ $t = 4s$ $s = 0.3m$ $a = ?$

$s = ut + \frac{1}{2}at^2$ $0.3m = (0ms^{-1})(4s) + \frac{1}{2}a(4s)^2$

$a = 0.0375 ms^{-2}$

$F = ma$ $\sin\frac{3}{5} = \frac{F_H}{0.5g\ N}$ $F_H = 0.0514N$

$0.0514N - TN = 0.5kg \times 0.0375ms^{-2}$

$TN = 0.0326N$

(b) Calculate the coefficient of friction between B and the horizontal surface. [5]

$F = ma \quad 0.0326N - F_r = 0.2kg \times 0.0375ms^{-2}$

$F_r = 0.025N$

$F_r = \mu R \quad R = 0.2kg \times 9.81\,ms^{-2} = 1.962N$

$0.025N = \mu \times 1.962N$

$\mu = 0.0128$

4 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle R of mass 2kg is moving on a smooth horizontal surface under the action of a single horizontal force $\mathbf{F}$N. At time t seconds, the velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ of R, relative to a fixed origin O, is given by $\mathbf{v} = (pt^2 - 3t)\mathbf{i} + (8t + q)\mathbf{j}$, where p and q are constants and $p < 0$.

(a) Given that when $t = 0.5$ the magnitude of $\mathbf{F}$ is 20, find the value of p. [6]

R: 2kg FN ts Vms^{-1}

$V = (pt^2 - 3t)\underline{i} + (8t + q)\underline{j}$

$F = ma \quad 20N = 2kg \times a \quad a = 10ms^{-2}$

$a = \frac{dv}{dt} = (2pt - 3)\underline{i} + (8)\underline{j} \Rightarrow 10\,ms^{-1}$

$(2p(0.5) - 3)\underline{i} + 8\underline{j} = (p-3)\underline{i} + 8\underline{j}$

$\sqrt{(p-3)^2 + 8^2} = 10$

$(p-3)^2 = 36$

$p - 3 = \pm 6$

$p = \cancel{9}$ or -3

X

as $p < 0$

$p = -3$

When $t = 0$, R is at the point with position vector $(2\mathbf{i} - 3\mathbf{j})$ m.

(b) Find, in terms of q, an expression for the displacement vector of R at time t. [4]

$$R = \int v \, dt = \int (-3t^2 - 3t)\underline{i} + (8t + q)\underline{j} \, dt$$

$$= \left(\tfrac{1}{3}(-3)t^3 - \tfrac{3}{2}t^2 + c\right)\underline{i} + \left(4t^2 + qt + c\right)\underline{j}$$

when $t = 0$

$$-(0)^3 - \tfrac{3}{2}(0)^2 + c = 2 \qquad c = 2$$

$$4(0)^2 + q(0) + c = -3 \qquad c = -7$$

$$R = \left(-t^3 - \tfrac{3}{2}t^2 + 2\right)\underline{i} + \left(4t^2 + qt - 7\right)\underline{j}$$

When $t = 1$, R is at a point on the line L, where L passes through O and the point with position vector $2\mathbf{i} - 8\mathbf{j}$.

(c) Find the value of q. [3]

$t = 1$

$$R = \left(-1 - \tfrac{3}{2} + 2\right)\underline{i} + (4 + q - 7)\underline{j}$$

$$= -\tfrac{1}{2}\underline{i} + (q - 3)\underline{j}$$

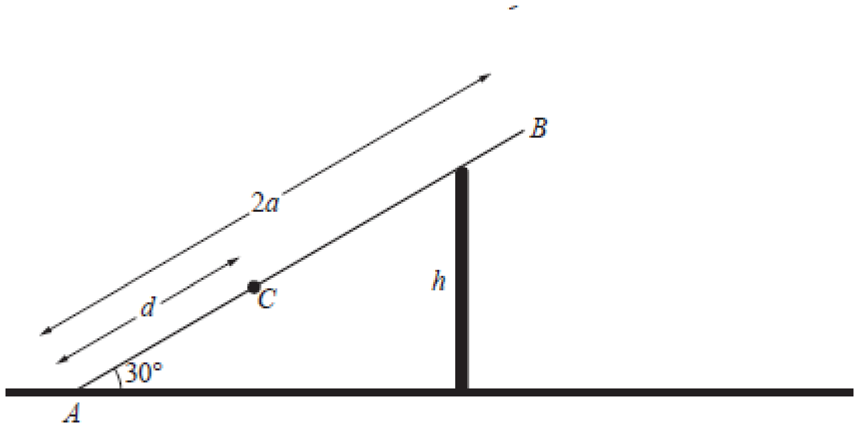
$(0,0)$ $(2, -8)$ $\qquad m = \dfrac{-8-0}{2-0} = \dfrac{-8}{2} = -4$

$(0,0)$ $(-\tfrac{1}{2}, q-3)$ $\qquad -4 = \dfrac{q-3-0}{-\tfrac{1}{2}-0}$

$$2 = q - 3$$

$$q = 5$$

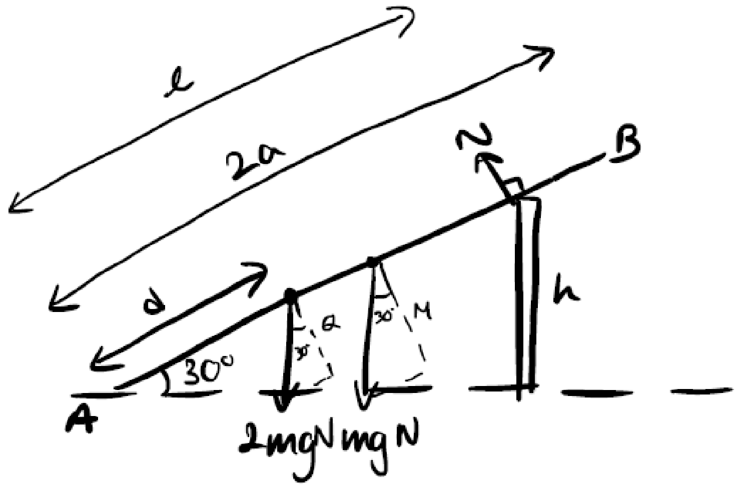
5



The diagram shows a ladder AB, of length $2a$ and mass m, resting in equilibrium on a vertical wall of height h. The ladder is inclined at an angle of 30° to the horizontal. The end A is in contact with horizontal ground. An object of mass $2m$ is placed on the ladder at a point C where $AC = d$.

The ladder is modelled as uniform, the ground is modelled as being rough, and the vertical wall is modelled as being smooth.

(a) Show that the normal contact force between the ladder and the wall is $\frac{mg(a+2d)\sqrt{3}}{4h}$. [4]



$\cos 30° = \frac{Q}{2mg}$ $\quad Q = \frac{\sqrt{3}}{2} \times 2mg\ N$
$\quad = \sqrt{3}\, mg\ N$

$\cos 30° = \frac{M}{mg}$ $\quad M = \frac{\sqrt{3}}{2} \times mg\ N$
$\quad = \frac{\sqrt{3}}{2} mg\ N$

$\sin 30° = \frac{h}{l}$ $\quad l = 2h$

CM: $\sqrt{3}\, mg\, N \times d\, m + \frac{\sqrt{3}}{2} mg\, N \times \frac{2a}{2} m$
$= \sqrt{3} d\, mg\ Nm + \frac{\sqrt{3}}{2} a\, mg\, Nm$
$= \frac{\sqrt{3}}{2}(a + 2d)\, mg\ Nm$

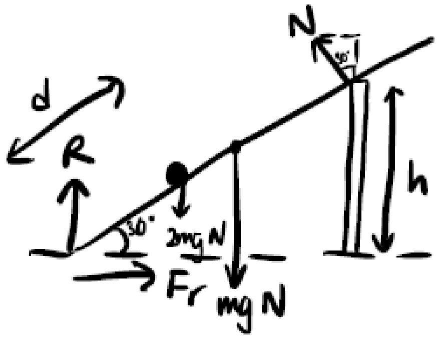
ACM: $N \times 2h\, m = 2Nh\ Nm$

$\frac{\sqrt{3}}{2}(a+2d)mg\ Nm = 2Nh\ Nm$

$$N = \frac{mg(a+2d)\sqrt{3}}{4h}$$

It is given that the equilibrium is limiting and the coefficient of friction between the ladder and the ground is $\frac{1}{8}\sqrt{3}$.

(b) Show that $h = k(a + 2d)$, where k is a constant to be determined. [7]



$$R + \cos 30° \times N = 2mg + mg$$

$$R + \frac{\sqrt{3}}{2}N = 3mg \qquad R = 3mg - \frac{\sqrt{3}}{2}N$$

$$R = 3mg - \frac{3}{8}\frac{mg}{h}(a+2d)$$

$$F_r = \sin 30° \times N = \frac{1}{2}N$$

$$F_r = \frac{\sqrt{3}}{8}\frac{mg}{h}(a+2d)$$

$$F_r = \mu R$$

$$\frac{\cancel{\sqrt{3}}}{\cancel{8}}\frac{\cancel{mg}}{h}(a+2d) = \frac{1}{\cancel{8}}\cancel{\sqrt{3}} \times \left(3\cancel{mg} - \frac{3}{8}\frac{\cancel{mg}}{h}(a+2d)\right)$$

$$\frac{1}{h}(a+2d) = 3 - \frac{3}{8h}(a+2d)$$

$$\left(\frac{1}{h} + \frac{3}{8h}\right)(a+2d) = 3$$

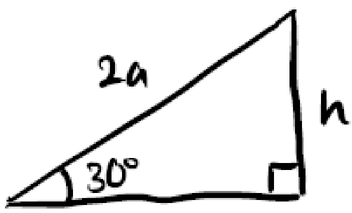
$$\frac{11}{8h}(a+2d) = 3$$

$$\frac{11}{24}(a+2d) = h$$

$$h = \frac{11}{24}(a+2d)$$

$$k = \frac{11}{24}$$

(c) Hence find, in terms of a, the greatest possible value of d. [2]



$\sin 30° = \frac{h}{2a}$

$\frac{1}{2} = \frac{h}{2a}$ $\quad h = a$

$\frac{11}{24}(a+2d) = a$

$11a + 22d = 24a$

$22d = 13a$

$d = \frac{13}{22}a$

(d) State one improvement that could be made to the model. [1]

consider the friction between the object & ladder

Total Marks for Question Set 4: 50 Marks